# MASTER PPD - PUBLIC ECONOMICS: TAX & TRANSFER POLICIES

FINAL EXAM, NOVEMBER 25, 2010, 2 P.M. - 4 P.M.

*The exam is 2 hours long.* ***No*** *document whatsoever is allowed.*

## 1. Exercise 1: Tax Revenues and Tax Incidence (5 points)

1) Define national income (0.5 point) and give the rough level of taxes in rich countries as a fraction of national income (0.5 point).

2) Give a rough breakdown of total tax receipts between consumption taxes, labor taxes and capital taxes, as of today in rich countries. (0.5 point)

3) Why is it difficult to know exactly who ends up paying taxes? What methods can shed light on this issue? (1 point)

4) According to macroeconomic data, who pays employer social contributions: capital, labor, both? Why? (1 point)

5) Give and present briefly an example of an incidence study using micro data. (1,5 point).

## 2. Exercise 2: Income Tax and Income Distribution (5 points)

For incomes earned in 2009, the U.S. federal income tax schedule for a single person was the following:

| Tax bracket | Tax rate |
|---|---|
| \$0 - \$8,375 | 10% |
| \$8,376 - \$34,000 | 15% |
| \$34,001 - \$82,400 | 25% |
| \$82,401 - \$171,850 | 28% |
| \$171,851 - \$373,650 | 33% |
| $>$ \$373,651 | 35% |

1) Write the formula that gives the average tax rate of a person with income of \$100,000. Is this average tax rate smaller, equal or bigger than 28%? (1 point)

2) How has the top marginal income tax rate evolved in the U.S. since the end of Word War II? (1 point)

3) Are the U.S. the only country where the top marginal income tax rate has experienced such an evolution? Give an example. (1 point)

4) What has been the trend in the share of aggregate pre-tax income accruing to top income groups in the United States since the beginning of the 1980s? (1 point)

5) Describe a mechanism by which lower top marginal tax rates have a causal effect on the observed share of total pre-tax income accruing to the top 1%. (1 point)

## 3. Exercise 3: Pigouvian Corrective Taxation (5 points)

Consider an economy with a continuum of agents $i$ in $[0, 1]$. There are two goods: a non-energy good and an energy good. Each agent has the same utility function:

$$U_i = U_i(c_i, e_i, E) = (1 - \alpha)log(c_i) + \alpha log(e_i) - \lambda log(E)$$

where:

- $c_i$ is the individual consumption level of the non-energy good,
- $e_i$ is the individual consumption level of the energy good,
- $E$ is the aggregate consumption level of the energy good,
- $0 < \alpha < 1$ and $0 < \lambda < 1$

1) Explain briefly what it means that $E$ enters negatively in the utility function $U_i$. (0.5 point)

2) In a laissez-faire economy, each agent $i$ chooses its level of consumption of the non-energy good $c_i$ and of the energy good $e_i$ to maximize its utility under the budget constraint $y_i = c_i + e_i$. Compute the optimal levels of $c_i$ and $e_i$. (1 point)

3) In a planned economy, a benevolent planner chooses the aggregate level of consumption of the non-energy good $C$ and of the energy good $E$ to maximize social welfare $U = U(C, E, E)$ under the aggregate budget constraint $Y = C + E$. Compute the socially optimal values of $C$ and $E$. (1 point)

4) Show and explain briefly why the consumption of energy good is lower in the planned economy than in the laissez-faire economy. (0.5 point)

5) Now we introduce a corrective tax $t$ on energy consumption. Agent's $i$ budget constraint becomes $c_i + (1+t)e_i = y_i$. Compute the optimal levels of $c_i$ and $e_i$. (0.5 point)

6) Compute the tax rate $t^*$ that allows to obtain the socially optimal value of consumption of the energy good. (1 point).

7) Show that $t^*$ is an increasing function of $\lambda$, and explain briefly what it means. (0.5 point)

## 4. Exercise 4: Optimal Taxation of Labor Income (5 points)

This exercise is based on a simplified version of the model presented in section II of Piketty T. (1995), "Social Mobility and Redistributive Politics", *Quarterly Journal of Economics*, vol. CX, 3, p. 551-584.

Consider an economy with a continuum of agents $i$ in $[0,1]$. A fraction $f_l$ of agents is lucky, a fraction $f_u$ is unlucky ($f_l + f_u = 1$). Each agent can obtain a high labor income $y_1$ or a low labor income $y_0$ ($y_1 > y_0$) depending on whether or not he is lucky, and on how much effort $e$ he makes.

More precisely, the probability for an unlucky agent making effort $e$ to have labor income $y_1$ is:

$$P[y_i = y_1 | e_i = e \text{ and } i \text{ is unlucky}] = \pi_0 + \theta e$$

the probability for a lucky agent making effort $e$ to have labor income $y_1$ is:

$$P[y_i = y_1 | e_i = e \text{ and } i \text{ is lucky}] = \pi_1 + \theta e$$

where $\pi_0 < \pi_1$ and $\theta > 0$.

1) Explain in one sentence what the assumption $\pi_0 < \pi_1$ means (0.5 point) and in one sentence what the parameter $\theta$ captures (0.5 point).

Let's introduce a redistributive tax system: all incomes are taxed at rate $\tau$ ($0 < \tau < 1$) and all tax revenues are redistributed in a lump-sum way. Therefore, the after-tax income of a person with low pre-tax income is $(1-\tau)y_0 + \tau Y$ (where $Y$ is aggregate income). The after-tax income of a person with high pre-tax income

is $(1-\tau)y_1 + \tau Y$. Each agent has the same utility function:

$$U_i = y_i - \frac{e_i^2}{2}$$

2) (0.5 point) Explain in one sentence why the expected utility of an unlucky agent is:

$$U_i = (1-\pi_0-\theta e)(1-\tau)y_0 + (\pi_0+\theta e)(1-\tau)y_1 + \tau Y - e_i^2/2 \tag{1}$$

3) Compute the effort level $e^*$ that maximizes the expected utility of an unlucky agent and interpret in one sentence the result. (1 point)

4) Compute the optimal effort level of lucky agents. (0.5 point)

5) Before the birth of any agent (i.e., before anyone knows if he is lucky or not), society decides to set the tax rate $t$ at the level that maximizes the expected utility of unlucky agents (eq. (1)), given the optimal effort level $e^*$ of lucky and unlucky agents. Society cannot observe luck nor effort.

a) (1 point) Show that the optimal tax rate is:

$$\tau = \frac{f_l(\pi_1 - \pi_0)}{(y_1 - y_0)\theta^2}$$

b) Interpret this formula. (1 point)