

Public Economics: Tax & Transfer Policies

Final Exam, February 18, 2016 - 2 hours

*The exam is 2 hours long and can be done either in French or English. **No** document whatsoever is allowed.*

1 Welfare theorem and taxation (4.5 points)

1) According to the first welfare theorem, what are the rationales for taxation ? (1.5 points)

2) According to the second welfare theorem, what should be a non-distortionary redistributive policy ? Why is it difficult in practice to implement such a policy ? (1.5 points)

3) How has evolved tax revenues in France and in the UK during the XXth century ? How could we explain the different levels of taxation between these two countries today ? (1.5 points)

2 Exercise 1 : Income inequality and redistribution (7.5 points)

Consider an economy made up of a continuum of agents i in $[0, 1]$. The utility function of an agent i is given by :

$$U_i = y_i - C(e_i)$$

$$\text{with } C(e_i) = \frac{e_i^2}{2a}, a > 0$$

Each individual i can obtain one of two possible pretax incomes $y_i = y_0$ or y_1 (with $y_1 > y_0 > 0$) depending on his/her ability. We note L the fraction of low-ability individuals, i.e individuals with ability θ_0 .

The probability for an individual with low ability making effort e to have labor income y_1 is :

$$P[y_i = y_1 | e_i = e, \theta = \theta_0] = \theta_0 e$$

The probability for an individual with high ability making effort e to have labor income y_1 is :

$$P[y_i = y_1 | e_i = e, \theta = \theta_1] = \theta_1 e$$

where $0 < \theta_0 < \theta_1$ and $\theta > 0$.

Let's introduce a redistributive tax system : all incomes are taxed at rate $0 < \tau < 1$ and all tax revenues are redistributed in a lump-sum way $\tau \cdot Y$, where Y is aggregate income at the corresponding period.

- 1) Express the after-tax income of a person with low or high pre-tax income. (1 point).
- 2) Express the expected utility of a person with low ability. (1 point).
- 3) Compute the effort level e^* that maximizes the expected utility of an individual with low ability and interpret the result. (1 point)
- 4) Compute the optimal effort level for an individual with high ability. (0.5 point).

Suppose that the government decides to set the tax rate t at the level that maximizes the expected utility of individuals with low ability. Note that the government can not observe directly the effort made by individuals.

- 5) Express the aggregate income Y as a function of the optimal effort level e^* . (1 point)

6) Compute the optimal tax rate. (2 points)

7) Explain why the optimal tax rate would be equal to zero if $\theta_1 = \theta_0$. What assumption should be changed in order to get a different result? (1 point)

3 Exercise 2 : Optimal capital taxation (8 points)

We consider a simple model of wealth accumulation with one period and a continuum of agents i in $[0, 1]$. At the beginning of the period, individuals work to earn exogenous lifetime earnings y_{Li} and receive an inheritance b_i^r . In this simple economy, there is no growth, no return on capital and no discount factor. There is no taxation on labor, nor taxation on inheritance received. For redistribution purpose, the government decides to introduce a wealth tax on wealth accumulated at the end of the first period in order to finance a lump-sum transfer. Therefore, the lifetime resources of individuals during the first period is equal to the sum of labor income y_{Li} , bequests received b_i^r and lump-sum transfer g for all individuals. The lump-sum transfer g is entirely financed by linear taxation on end-of-period wealth such as $g = \tau_w \cdot w$, with w the average end-of-period wealth and τ_w , the tax rate on wealth. At the end of the period, individuals split their resources into consumption c_i and wealth accumulated w_i . They derive utility over consumption and after-tax wealth (partly because of bequest motives and partly because wealth brings utility per se. The utility function is given by :

$$V_i = U_i(c_i) + \phi_i((1 - \tau_w)w_i)$$

We take as given the joint distribution $f(y_{Li}, b_i^r, U_i, \phi_i)$ of labor income, bequests received and preferences over consumption and wealth.

1) Define the budget constraint and the the maximisation program of an individual i . (2 points)

2) What is the first order condition for wealth accumulation w_i ? (2 points)

3) The government wants to define wealth tax rate τ_w in order to maximize the welfare of individuals. The average wealth w is a function of $(1 - \tau_w)$. We define the aggregate elasticity of wealth to wealth tax such as $e_w = \frac{1-\tau_w}{w} \frac{dw}{d(1-\tau_w)}$. The aggregate elasticity e_w comes from the aggregation of all individual maximisation programmes.

Show that the wealth tax rate maximizing the utility of a given individual i is equal to $\tau_w = (1 - w_i/w)/(1 + e_w)$? (2 points)

4) Interpret the formula (2 points)

5) Bonus question : Now assume that the government wants to maximise the sum of utilities of all individuals with zero bequest received, possibly with different weights ω_i in order to normalize for preference heterogeneity. The social welfare function of the government is given by :

$$SWF = \int_{i \in [0,1] s.t. b_i^r = 0} \omega_i [U_i(y_{Li} + \tau_w \cdot w - w_i) + \phi_i((1 - \tau_w)w_i)]$$

What would be the optimal tax rate? (3 points)