

Master APE - Economics of Inequalities

Final Exam, January 28, 2014 - 14h00-16h00. No document allowed.

Exercise 1: Income Inequalities (5 points)

We consider a very simple framework where the production function of an economy is defined by :

$$Y = F(L_s, L_u) = L_s^\alpha \cdot L_u^{1-\alpha}$$

w_s and w_u are respectively the prices of the high-skill labor L_s and the low-skill labor L_u . The price of the output Y is set to one.

1) What is the implication of the Cobb-Douglas function choice on the skilled and unskilled labor shares ? (1.5 points)

A: The choice of a Cobb-Douglas function implies that the skilled and unskilled labor shares are entirely set by technology (α) and do not depend on quantities L_u and L_s . Indeed, the elasticity of substitution between L_u and L_s is equal to 1. If skilled wage rises by 1%, then firms use 1% less skilled labor, so that skilled labor share in total labor share remains the same as before.

2) Determine the relative wage of high-skill labor $\frac{w_s}{w_u}$ (1 point). A:

$$\text{Max } \pi = F(L_s, L_u) - w_s \cdot L_s - w_u \cdot L_u$$

$$\text{FOC: } \frac{w_s}{w_u} = \frac{L_u}{L_s} \cdot \frac{\alpha}{1-\alpha}$$

3) How would this analysis change with a CES production function ? Discuss the economic intuitions and the policy implications. (1 point)

A: CES function :

$$Y = F(L_s, L_u) = [aL_s^{(\sigma-1)/\sigma} + bL_u^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)}$$

where σ = constant elasticity of substitution between L_s and L_u .

The economic intuition and the policy implications depends on the value of σ . When $\sigma \rightarrow \infty$, there is an infinite substitution between L_s and L_u . The ratio $\frac{w_s}{w_u}$ reflects only the difference in productivity between the skilled and the unskilled workers.

When $\sigma \rightarrow 1$, the CES function converges toward the Cobb-Douglas function.

When $\sigma \rightarrow 0$, there is no substitution possibility between L_s and L_u .

4) According to this model, what is the role that market forces may have played in the evolution of income inequalities in the USA since the 1970s ? (1.5 points)

A: In this model, wage inequalities remain stable only if the skilled labor level (L_s) increases at the same rate as that required by technical change (α). On the contrary if the skilled labor force is not sufficient to respond to the technical change (higher α) then wage inequalities rise.

5) According to you, how could the institutions reduce or limit the evolution of pre-tax income inequalities ? (quote at least two factors) (1 point)

A: massive investment in skills (by increasing access to higher education) or labor market institutions (salary scales and minimum/maximum wages)

Problem : Wealth Inequalities (10 points)

Part 1 : Questions (5 points)

1) Describe the evolution of wealth concentration during the 20th century in France. (1 point)

A: In 1914, top 10% wealth share = 80%. Sharp decline from 1914 to 1970. Then wealth concentration rises since 1970-1980s but it is still much lower in the 2010s than in 1910s.

2) According to you, what are the factors that could explain this evolution ? (1 points)

A: The sharp decline could be explained by the two world war adverse shocks. The non catch-up of the 1910s concentration level could be explained by the introduction of progressive taxation on income and wealth.

3) According to the pure lifecycle model of Modigliani, what is the rationale for wealth inequalities ? (1.5 points)

A: In this model, wealth accumulation is entirely driven by life-cycle motives (i.e. savings for retirement). Wealth inequalities are due to different positions of the agents in the life-cycle : agents save during their working life; during retirement, they consume their past saving and die with zero wealth.

4) According to the pure dynastic model with two class of dynasties, what is the rationale for wealth inequalities ? (1.5 points)

A: In the pure dynastic model, there are two class of dynasties : High-wealth dynasties and Low-wealth dynasties. Individuals maximize dynastic utility functions, as if they were infinitely lived; death is irrelevant in their wealth trajectory, so that they die with positive wealth. This framework implies that everybody works the same, but some dynasties are permanently richer and consume more.

Part 2 : The random-shocks model (5 point)

Consider an individual from a dynasty i and generation t that maximizes the following utility function:

$$V_{it}(c, w) = c_{it}^{1-s_{it}} \cdot w_{it+1}^{s_{it}}$$

In this framework, w_{it} represents the wealth received from the previous generation and w_{it+1} , the wealth left to the next generation. For simplicity, we suppose that each individual i receives the same labor income $y_{Lit} = y_{Lt}$ and the same annual rate of return $r_{it} = r_t$. Thus, the individual allocates his total resources

$y_{Lt} + (1 + r) \cdot w_{it}$ between consumption c_{it} and end-of-life wealth w_{it+1} .

1) What is the budget constraint of an individual i at time t . (1 point)

$$A: y_{Lt} + (1 + r) \cdot w_{it} = c_{it} + w_{it+1}$$

2) Determine the optimal level of consumption c_{it}^* (1 point)

$$A: V_{it}(c, w) = c_{it}^{1-s_{it}} \cdot w_{it+1}^{s_{it}}$$

From the budget constraint expression, we obtain:

$$\Rightarrow V_{it}(c, w) = c_{it}^{1-s_{it}} \cdot (y_{Lt} + (1 + r) \cdot w_{it} - c_{it})^{s_{it}}$$

$$FOC: c_{it}^* = (1 - s_{it}) \cdot (y_{Lt} + (1 + r) \cdot w_{it})$$

3) Determine the individual-level transition equation for wealth : w_{it+1}^* (1 point)

By replacing the expression of c_{it}^* in the budget constraint, we obtain :

$$w_{it+1} = s_{it} \cdot [y_{Lt} + (1 + r)w_{it}]$$

3)What is the meaning of the parameter s_{it} ? (1 point)

A: s_{it} is the saving taste parameter. It reflects the taste for bequest relatively to consumption.

4) Compare the rationale for wealth inequalities in this model and in the pure dynastic one ? (1 point)

A: In this model, the wealth of an individual depends on the different tastes for bequest from the previous generations of the same dynasty. Therefore, the wealth inequalities can be explained by the differences in taste among each generation and dynasty. Contrary to the dynastic model, this model allows a positive wealth mobility because of the randomness of the bequest taste among generation and dynasty.

Exercice 2 : Capital-income ratio (5 points)

1) How has the capital-income ratio evolved during the 20th century in Europe and in the USA ? (1.5 points)

A: The national wealth-income ratio has followed a large U-shaped curve in Europe during the 20th century: 600-700% until 1910, down to 200-300% around 1950, back to 500-600% in 2010. The capital income ratio is almost stable all along the 20th century for the USA around 400%-500%.

2) According to you, what are the factors explaining the different evolutions of the capital-income ratio in the United Kingdom and in the USA during the period 1910-1950.

A: The capital-income ratio was lower in USA than in Europe in 1910 : the American had less time to accumulate capital and the land was cheaper. A significant part of the UK wealth was due to large net foreign assets, which disappeared during the period and never reappeared. The two wars can have played a role more important in the UK than in the USA.

3) How can the Harrod-Domar-Solow steady-state formula explained the variation of the capital-income ratio during the period 1970-2010 ? (2 points)

A: H-D-S Formula : $\beta = s/g$. Period 1970-2010 is characterized by a growth slowdown in the developed country as compared to the saving rate leading to a rise of the capital-income ratio.