From Classes to Copulas: Wages, capital, and top incomes[1]

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Introduction

For classical economists, there was a straightforward relationship between the factor distribution of income and the distribution of income among persons. There were workers, capitalists and landlords as separate classes, receiving wages, profits and rent, respectively. Workers were assumed to be at the bottom of the ladder, and a rise in the wage share reduced inequality in the personal distribution. A rise in the share of investment income – combining profits and rent – increased inequality in the personal distribution

At the beginning of the twentieth century, there remained strong elements of the class system, in that top incomes were made up predominantly of investment income. Piketty and Saez show that in the United States (US) in 1916 the capital income share for the top 0.5 per cent was over 50 per cent, and that the share of earned income for the top 0.1 per cent was only 10 per cent (2007, Table 5A.7). As they observe, "top corporate executives at the beginning of the century were only a tiny minority within the top taxpayers" (2007, page 152). In the United Kingdom (UK) in 1911 investment income made up 72.3 per cent of the income of those assessed to super-tax (Atkinson, 2007, page 109).

This has now changed. Over the twentieth century there was in the US a "dramatic evolution of the composition of top incomes" (Piketty and Saez, 2007, page 152). There has been a "surge" in top wage earnings, and the "working rich" are now to be found in the top income ranges, along with the top capital owners ("rentiers") who populated the top 1 per cent in earlier times. According to Wolff and Zacharias, "the two groups now appear to co-habitate the top end of the income distribution (2009, page 108). In France, (Piketty 2003) found that the top capital incomes had not been able to recover from a succession of adverse shocks over the period 1914 to 1945; progressive income and inheritance taxation had prevented the re-establishment of large fortunes.

The aim of this paper is to explore further the changing composition of top incomes, by examining in greater depth the roles of earned (labour) and investment (capital) income. Not only will this two-way decomposition help link the changes in top shares to macro-economic developments (the changes in the wage share), but also it will aid our understanding of the wider social implications of distributional change. The substantial rise in top income shares that has taken

[1] Emmanuel Saez participated in the first stages of this project, and we should like to thank him for his assistance with the US data.

place in many (but not all) advanced countries means that it is important to understand the underlying mechanisms. Are the two groups remaining separate, or are we witnessing a fusion of top capital and top earnings? Is one class simply rising while another falls, or are the classes merging? Put differently, can the change in the marginal distribution of earnings on its own explain the rise in top shares? Are top executives and financiers simply elbowing capital owners out of the top group? If that is the case, then it is the distribution of earnings that should have first claim on our attention. But if those at the top are increasingly receiving income from both sources, then we have to pay greater attention to the ownership and transmission of wealth.

In the first section of the paper, we set out the analytical framework. We make use of known results, but the key concept of the copula function has not been applied to the problem at hand. As is explained, we do not seek to estimate parametric copula functions but rather to compare the degree of association, proposing a straightforward procedure for the implementation of dominance criteria.

In undertaking this analysis of the composition of top income shares, we are fully aware of the limitations of the data. The income tax data on which the above conclusions are based do not adequately capture the full return to capital. Moreover, the extent of coverage has fallen over time as there has been erosion of capital income from the progressive income tax base. At the outset a number of income tax systems (such as those of France or the United Kingdom) included imputed rents of homeowners in the tax base, but today imputed rents are typically excluded. Where the tax base has been extended, this has in some cases taken the form of separate taxation (as with capital gains in the UK), so that the income is not covered in the income tax data. As a result of these developments, the share of capital income that is reportable on income tax returns, and hence included in the series presented, has significantly decreased over time. For this reason, we have focused on two countries where the data are less incomplete: the US, where the data include capital gains, and Norway, where estimates have been made of Hicksian measures of capital income (Aaberge and Atkinson, 2010). The data are described in section 2.

1. Analytical framework

We are concerned with the decomposition of total personal income into two components: earned (labour) income and investment (capital) income. Personal investment income is derived from profits and rents (as well as from interest paid on government and other debt), and from part of self-employment income (the remainder is attributed to earned income – see below).

With this two-way division, the personal distribution of total income depends on four elements:

a) The shares of earned and investment income (which add to 100 per cent);
b) The marginal distribution of earned incomes;
c) The marginal distribution of capital incomes;
d) The correlation between earned and capital incomes.

The developments in top shares described earlier have highlighted a number of these elements. In the US context, it is natural to suppose that a substantial fraction of the rise in top incomes was due to a surge in top wage incomes (mechanism (b)). The decline of the rentier also reflects reduced concentration of wealth (mechanism c). Both marginal distributions have moved in ways that shift the composition of top incomes towards earned income. But the other two elements are potentially important. There has in recent years been a shift towards an increased share of capital income. This is one factor that has been identified in studies of top incomes in Nordic countries as having operated in the direction of restoring the role of capital income. A study of Finland concluded that 'the main factor that has driven up the top 1 per cent income share in Finland after the mid 1990s is an unprecedented increase in the fraction of capital income' (Jantti et al. 2010). In Sweden, Roine and Waldenström (2008) report that "between 1945 and 1978 the wage share at all levels of top incomes became more important . . . But in 2004 the pattern is back to that of 1945 in terms of the importance of capital, in particular when we include realized capital gains".

In contrast to the elements (a) to (c), the fourth mechanism – the pattern of association - has received very little attention. Yet this is potentially important. The observed change in the composition of the top income group may also result from changes in the correlation. There may no longer be a sharp distinction between workers and rentiers (capitalists). In the pure class model, the correlation between labour and capital income is minus 1. The correlation may now be greater than this, and may indeed be positive.

It is therefore tempting to measure this fourth element in terms of the correlation. The Pearson correlation coefficient is not however well-suited for this purpose, since it is not independent of changes in the marginal distributions. Suppose, for example, in the class model, workers are divided into two sub-classes, with the same mean wage, but with one class earning β^2 as much as the other. An increase in β means that the correlation coefficient moves away from minus 1, but there is no change in rankings or in the composition of the top income group. On the other hand, this objection does not apply to the rank correlation and in this paper we consider the cross-correlation in terms of the *copula function*, which provides a clean way of isolating changes independent of changes in the marginal distribution.

Proceeding more formally, let us denote labour income by ℓ and capital income by k, where $\ell \geq \underline{\ell}$ (>0) and $k \geq \underline{k}$ (>0). The marginal distributions of, respectively, earned income and capital income are denoted by $F(\ell)$ and $G(k)$, and each is defined in proportionate terms, so that both tend to 1 as income

approaches infinity. Since we are interested in the top incomes, we consider the survival distributions F^(ℓ) = 1-F(ℓ), the proportion of the population with earned income of ℓ or higher, and G^(k) = 1-G(k), the proportion with capital income of k or higher. The joint distribution of *k* and ℓ is denoted by *H(k,ℓ)*, and the joint survival function by *H^(k,ℓ)*.

The copula is the function that binds together the two marginal distributions. Or rather, since our purpose is to study the top shares, it seems better to use the *survival copula* C^{F^,G^} (see Nelsen, 2006, pages 32-33). This shows the proportion of the population whose rank is F^ or higher in terms of labour income and G^ or higher in terms of capital income. By Sklar's Theorem (Nelsen, 2006, page 18),

$$H^(\ell,k) = C^{F^(\ell),G^(k)} \qquad (1)$$

is the joint survival function for (ℓ,k). In other words, we can obtain the survival copula from the survival function by substituting ℓ(F^) and k(G^). It should be noted that the copula is invariant with respect to strictly increasing transformations of these functions. The properties of the copula function are described clearly by Dardanoni and Lambert (2001) in their analysis of the measurement of horizontal equity.

The attraction of the copula is that it allows us to separate cleanly the changes in the relative rankings of individuals from changes in the relative weight of the two sources of income and from changes in the marginal distributions. Suppose that there is a shift away from capital income towards labour income, in such a way that all capital income components are reduced proportionately and all labour incomes are increased proportionately. This leaves the ranks in each dimension unchanged. The copula function is therefore unchanged.

The implementation of the copula may proceed parametrically or non-parametrically. A number of specific functional forms have been proposed for C^ - see Nelsen (2006, Table 4.1) for one-parameter families. For example, there is the bi-variate Pareto (see Hutchinson and Lai, 1990, and Nelsen, 2006, page 33), where

$$(C^)^{-\gamma} - 1 = [(F^)^{-\gamma} - 1] + [(G^)^{-\gamma} - 1] \qquad (2)$$

where $-1 \leq \gamma$. So that C^{1,G^} = G^, and C^{F^,1} = F^, and C^{0,G^} = C^{F^,0} = 0. Just as with the constant elasticity of substitution production function, the parameter γ allows the shape of the contours to vary. This family is said to be comprehensive in that it includes the cases where C^ is simply the zero correlation case of the product F^G^, which is obtained as the limit as γ approaches 0, and where C^ approaches the upper and lower Fréchet-Hoeffding bounds. The latter are given by γ approaching infinity (correlation 1) and γ = -

1 (correlation of -1). Kendall's tau is equal to $\gamma/(\gamma+2)$. The estimated value of γ therefore provides a natural way of summarizing the degree of association between different income sources. In the Ricardian class model, γ is close to minus 1, whereas if γ is positive, then earned and capital incomes tend to be positively associated.

First-degree dominance

Parametric forms do however impose considerable structure on the joint distribution, notably symmetry, and we have therefore decided to proceed non-parametrically. One advantage is that such an approach is closer to that adopted in studies of social mobility, where a distinction is drawn between structural mobility and exchange mobility (holding the marginal distributions constant). Exchange mobility is studied directly in terms of transition matrices $[p_{ij}]$ where the cells are defined in terms of percentile position. The counterpart of such a matrix in the present application – the association matrix - is illustrated in Table 1 for the US in 2000 (the data are described further in the next section). The table shows for example that of the top 0.5 per cent in the wage distribution, 0.13 (or 26 per cent) were also in the top 0.5 per cent of the capital income distribution. Of the top 0.5 per cent in the wage distribution, 0.30 (or 60 per cent) were in the top 5 per cent of the capital distribution; whereas, of the top 0.5 per cent in the capital income distribution, a smaller number 0.26 (or 52 per cent) were in the top 5 per cent of the capital distribution. This underlines the need to allow for asymmetry.

We are interested in how far societies have moved from having a negative diagonal pattern to the transition matrix, as with the class society, to a situation where the two sources of income are independent or positively correlated. In order to assess such a movement, it is helpful to move from the frequencies to the cumulative distribution, as with the copula C^{F^,G^}. The survival function is shown for the US example in Table 2. This shows, for example, that all of the top 0.5 per cent of the wage distribution were in the top half of the distribution by capital income, whereas 0.11 (22 per cent) of the top 0.5 per cent of the capital income distribution were in the bottom half of the distribution by wage income.

Such a cumulative distribution can be used to compare the degree of diagonality. If we define a diagonalizing switch as one that adds and subtracts δ from adjacent cells in the frequency matrix $[p_{ij}]$ as follows:

$$\begin{matrix} P_{i,j} + \delta & p_{i,j+1} - \delta \\ P_{i+1,j} - \delta & P_{i+1,j+1} + \delta \end{matrix} \qquad (3)$$

(where i denotes the i-th percentile group, where i is counted from the top). The effect of the diagonalizing switch is to raise the survival copula by δ at (i+1,j+1) and to leave it elsewhere unchanged (in particular, the marginal distributions are unchanged) - see Atkinson (1981). On this basis, one distribution is closer to a positive diagonal (further from a class distribution) if its survival copula is everywhere higher or no lower. If, as we do below, we compare Table 2 for the US with the same table for earlier years, then this provides a simple dominance test as to the effect of the fourth mechanism: the changing degree of association between incomes from different sources.

Second-degree dominance

The test just described is one for first-degree dominance, and there may be situations in which this does not allow matrices to be ranked. When we compare the survival copulas for two dates, we may find that there are both positive and negative differences. As with one dimensional inequality measurement, the dominance criteria can be extended to second and higher degrees. In that case, the second-degree condition is obtained by integrating the cumulative distribution, which leads to a readily implementable test in terms of comparing Lorenz curves. In the present case, the second-degree dominance condition can be obtained by integrating the copula function, and this leads to a readily implementable test in terms of comparing rank correlations. As is noted by Nelsen (2006, page 170), the volume under the graph of the copula, when scaled appropriately, gives Spearman's rho measure of association. Such a procedure was suggested in Atkinson (1981) for the measurement of mobility. This involved the construction of the cumulated association matrices. We may contrast the integration of the copula (or, here, the survival copula) over (F,G) with the integration of $H(k,\ell)$ over (k,ℓ). The latter integration leads to the Pearson correlation coefficient (see Schweizer and Wolff, 1981, page 879),[2] which we have already rejected on the grounds that it is not independent of changes in the marginal distributions.

The procedure proposed here is to compare survival copulas in terms of the incomplete Spearman's rho coefficients over sub-intervals of F^ and G^ commencing at the origin (i.e. F=1 and G=1). There is dominance where the integrated copula is higher, or no lower, at all points. Comparing *survival* copulas is parallel to the concept, in one dimension, of "downwards Lorenz dominance" in Aaberge (2009). It means in effect that we are attaching greater weight to diagonalizing switches nearer the top of the distribution. If there are switches of type (3) at (i,j) and, in the reverse direction but equal amount, at (s,t), where s > i and t > j, then the integrated copula is raised – see Figure 1.

[2] Use of the incomplete co-variance is discussed in Atkinson and Bourguignon (1982, section 4).

2. Data

In this paper, we use individual tax return micro data for the United States and for Norway. The data are described for each country in turn.

United States

The data, on which we have worked in conjunction with Emmanuel Saez, are from the Internal Revenue Service (IRS) files from 1960 to 2005. These data are repeated cross-sections and they are publicly available since 1960. The data over-sample tax returns with high incomes and thus enable us to look at small fractiles at the top of the income distribution. In the analysis, sampling weights are employed to account for the over-sampling. The version of the data set used incorporates some minor adjustments, as documented in (Piketty and Saez, 2007). The unit of analysis is a tax return. Married couples in the US almost always file a joint tax return: in 1998, approximately 1 per cent of married women filed a separate return (Piketty and Saez, 2007, page).

In ranking taxpayers, we have employed population control estimates for the total number of tax units, as in Piketty and Saez (2007 updated on the website of E Saez). During the period considered, the fraction of filers lies between 88 per cent and 96 per cent. Non-filers are assumed to be located at the bottom of the distribution.

Labour income is defined as the sum of employment income and pension income. To this is added two-thirds of self-employment income, defined as income reported in Schedule C of the IRS tax form plus partnership income. The choice of two-thirds is arbitrary but does not seem unreasonable. Capital income is defined as the sum of income from dividends, interest (taxable and exempted), rents, estates and trusts, royalties, S-corporations,[3] capital gains and one-third of self-employment income.

Norway

In order to be more comparable with the calculations for the US, the Norwegian data are based on household rather than individual income, being based on the sum of the incomes of all individuals in the household. The data are from the micro data files of Statistics Norway and cover the years from 1993to 2009.

[3] An S-corporation is not subject to federal income tax. All incomes and losses are passed on to the owners of the corporation, who declare them on their individual tax returns.

The control total for the number of households is taken from the tax files <check>.

As with the US, labour income is defined as including two-thirds of self-employment income. Any individuals with a negative reported income from at least one of these three income sources are excluded from the analysis.

3. Results for the United States

We have already shown in Table 1 the association matrix for the United States in the year 2000. The degree of association appears strong. If we sum the entries for the top two groups, to look at the top 1 per cent, then over half of the top 1 per cent of capital income recipients find themselves in the top tenth of earners – see the extract from the association matrix in Table 3. Over a quarter of these taxpayers are in the top 1 per cent for both. Also, as noted earlier, the matrix is asymmetric. Over 60 per cent of the top 1 per cent of wage income recipients find themselves in the top tenth of capital income. Put the other way round, only 2 per cent of those in the top 1 per cent of wage earners find themselves in the bottom half of the capital income distribution. Being well paid seems to almost guarantee being well placed in terms of capital income. In contrast, 25 per cent of those in the top 1 per cent of capital income are located in the bottom half of the wage income distribution. One in 4 is close to being old-style capitalists. There is positive association but the US in 2000 is some distance from complete alignment of wage and capital incomes.

How is the extent of association changing over time? The numbers in brackets in Table 3 are the corresponding figures for 1980. These are all smaller than their counterparts for 2000. The degree of association increased between 1980 and 2000: in the former year only 17 per cent were in the top 1 per cent for both. The proportion of the top 1 per cent of earners who were in the top 10 per cent of capital income had been under a half in 1980, and rose from 47 per cent to 63 per cent. The rise for the corresponding percentage for the top 1 per cent of capital receivers was larger: from 32 per cent to 52 per cent.

Can we go further and conclude that the full matrix shows greater association in 2000? Table 4 shows the difference between the survival matrix in 2000 and that in 1980, where a positive entry implies that the (inverse) cumulative distribution is greater in 2000 than in 1980. Not all entries are positive, so that we do not have complete dominance. However, the negative entries (ignoring those due to rounding error) are only two and are outside the top 20 per cent.

4. Comparison of United States with Norway

The comparison with Norway is of interest since Norway, like other Nordic countries, is widely believed to be more egalitarian than the United States. The top income shares in Norway are considerably lower: in 2007 the share of the top 1 per cent was some half of that in the US (Aarberge & Atkinson, 2010 ?). Little is known however about the degree of association between wage and capital income. Is Norway less like a class society?

In Table 5, we show the same simplified version of the association matrix as in Table 3, except that the comparison is now being made between the US in and Norway for the year 2000. These figures suggest that the degree of association is less in Norway at the top of the income distribution. Of those in the top 1 per cent of capital income, 45 per cent were in the top 5 per cent of earned income in the US, compared with 30 per cent in Norway. Of those in the top 1 per cent of wage income, 50 per cent were in the top 5 per cent of capital income in the US, compared with 31 per cent in Norway. Indeed only less than a third (57 per cent) of this group in Norway were in the top 20 per cent of capital income receivers, compared with four-fifth in the US. There was more differentiation by class of income in Norway.

At the same time, Table 5 also shows that, if we enlarge the group studied to the top 20 per cent of wage earners, then we find that essentially the same percentage are in the top 1 per cent of capital income receivers. The very top may therefore look different in Norway. Table 6 shows the differences in the full survival matrices for the US and Norway. Positive entries correspond to wage and capital income being more correlated in the US. This is the case for the top 10 per cent of wage earners, but once we move below this level there are negative entries. For example, out of the top 20 per cent of wage earners, 12.0 per cent were in the top 5 per cent of capital income receivers in Norway (*not shown*), compared with 11.4 (2.27 / 10) per cent in the US.

We have seen that in the US the degree of association has tended to increase over time. Has the same happened in Norway? The Norwegian data go back to 1993, and they suggest that since then the proportion of the top 1 per cent for both did not rise (in fact it fell from 21 per cent in 1993 to 15 per cent in 1999). It has in fact remained close to the 1980 level for the US.

Table 7 shows the differences in the survival matrices for Norway for the full period from 1993 to 2009. These indicate that the tendency has been for the degree of association to become less at the very top (the shaded cells). Once we leave the top 1 per cent, of both wage and capital income, then the entries become positive, suggesting a greater positive association.

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Table 1 Joint frequency distribution of capital and wage income US 2000

	Capital income								
Wage income	Up to P50	P50-P59	P60-P79	P80-P89	P90-P94	P95-P99	P99-P99.5	P99.5-P100	*TOTAL*
Up to P50	30.09	2.90	8.13	4.90	2.18	1.54	0.14	0.11	*50*
P50-P59	6.10	0.96	1.53	0.70	0.43	0.24	0.02	0.01	*10*
P60-P79	9.61	2.77	4.29	1.73	0.95	0.56	0.06	0.04	*20*
P80-P89	3.13	1.83	2.88	1.02	0.58	0.47	0.05	0.03	*10*
P90-P94	0.85	0.97	1.65	0.74	0.34	0.39	0.04	0.03	*5*
P95-P99	0.31	0.45	1.35	0.74	0.39	0.57	0.09	0.09	*4*
P99-P99.5	0.01	0.02	0.10	0.10	0.06	0.11	0.05	0.04	*0.5*
P99.5-P100	0.00	0.01	0.05	0.07	0.07	0.12	0.05	0.13	*0.5*
TOTAL	*50*	*10*	*20*	*10*	*5*	*4*	*0.5*	*0.5*	*100*

Note: the numbers may not add up exactly on account of rounding.
Reading: of the 50 per cent of taxpayers who were in the bottom half of the wage distribution, 30.09 were in the bottom half of the capital income distribution: i.e. 60.18 per cent of them.

Table 2 Survival function for capital and wage income US 2000

2000	**Capital income**							
Wage income	Whole	Top half	Top 40 per cent	Top 20 per cent	Top 10 per cent	Top 5 per cent	Top 1 per cent	Top ½ per cent
Whole	100.00	49.90	39.99	20.00	10.00	5.00	1.00	0.50
Top half	50.00	29.98	22.98	11.11	6.02	3.20	0.75	0.39
Top 40 per cent	40.00	26.09	20.04	9.71	5.32	2.93	0.71	0.37
Top 20 per cent	20.00	15.69	12.42	6.37	3.71	2.27	0.61	0.33
Top 10 per cent	10.00	8.82	7.38	4.22	2.57	1.71	0.52	0.29
Top 5 per cent	4.99	4.67	4.19	2.68	1.78	1.26	0.45	0.26
Top 1 per cent	1.00	0.98	0.96	0.80	0.63	0.50	0.27	0.17
Top ½ per cent	0.50	0.50	0.49	0.44	0.37	0.30	0.18	0.13

Note: the numbers may not add up exactly on account of rounding.

Reading: of the top 0.5 per cent of taxpayers in the wage distribution, 0.37 were in the top 10 per cent of the capital income distribution: i.e. 74 per cent of them.

Table 3 The changing relation between earned income and capital income in US: 2000 (and 1980 in brackets)

	Capital Income			
Earned income	Top 1%	Top 5%	Top 10%	Top 20%
Top 1%	27 (17)	50 (36)	63 (47)	80 (68)
Top 5%	45 (27)			
Top 10%	52 (32)			
Top 20%	61 (38)			

Note: table reads as follows: of those in the top 1 per cent of capital income, 61 per cent were in the top 20 per cent of earned income in 2000 (compared to 38 per cent in 1980).

Table 4 Difference in survival function for capital and wage income US 2000 compared with 1980

	Whole	Top half	Top 40 per cent	Top 20 per cent	Top 10 per cent	Top 5 per cent	Top 1 per cent	Top 1/2 per cent
Whole	0.00	-0.08	0.00	0.00	0.00	0.00	0.00	0.00
Top half	0.00	-0.13	-0.63	1.19	1.21	0.78	0.24	0.12
Top 40 per cent	0.00	0.34	-0.25	1.27	1.25	0.78	0.24	0.12
Top 20 per cent	0.00	0.68	0.20	1.11	1.10	0.75	0.23	0.12
Top 10 per cent	0.00	0.56	0.32	0.94	0.85	0.65	0.21	0.12
Top 5 per cent	-0.01	0.26	0.26	0.59	0.59	0.49	0.18	0.11
Top 1 per cent	0.00	0.03	0.04	0.13	0.16	0.15	0.09	0.06
Top 1/2 per cent	0.00	0.01	0.01	0.06	0.08	0.08	0.05	0.05

Notes: (1) the numbers may not add up exactly on account of rounding; (2) the shading shows the cells that are negative by more than rounding error.
Reading: of the top 0.5 per cent of taxpayers in the wage distribution, 0.05 *more* were in the top 0.5 per cent of capital income receivers in 2000 than in 1980.

Table 5 The relation between earned income and capital income in US 2000 and Norway 2000 compared: Norway in brackets

	Capital Income			
Earned income	Top 1%	Top 5%	Top 10%	Top 20%
Top 1%	27 (15)	50 (31)	63 (41)	80 (57)
Top 5%	45 (30)			
Top 10%	52 (41)			
Top 20%	61 (55)			

Note: table reads as follows: of those in the top 1 per cent of capital income, 61 per cent were in the top 20 per cent of earned income in 2000 (compared to 55 per cent in Norway).

Table 6 Difference in survival function for capital and wage income US 2000 compared with Norway 2000

	Whole	Top half	Top 40 per cent	Top 20 per cent	Top 10 per cent	Top 5 per cent	Top 1 per cent	Top 1/2 per cent
Whole	0.00	-0.10	-0.01	0.00	0.00	0.00	0.00	0.00
Top half	0.00	2.96	1.58	-0.49	-0.91	-0.71	-0.01	0.00
Top 40 per cent	0.00	2.80	1.44	-0.49	-0.83	-0.68	0.00	0.01
Top 20 per cent	0.00	2.25	1.50	0.34	0.00	-0.12	0.06	0.05
Top 10 per cent	0.00	1.33	1.11	0.60	0.30	0.17	0.11	0.08
Top 5 per cent	-0.01	0.63	0.72	0.57	0.41	0.29	0.15	0.10
Top 1 per cent	0.00	0.09	0.14	0.23	0.23	0.19	0.11	0.08
Top 1/2 per cent	0.00	0.04	0.06	0.12	0.13	0.11	0.07	0.06

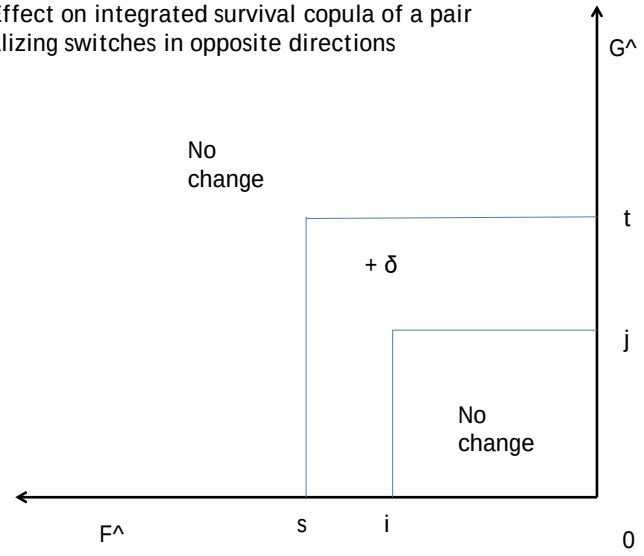
Notes: (1) the numbers may not add up exactly on account of rounding; (2) the shading shows the cells that are negative by more than rounding error.
Reading: of the top 0.5 per cent of taxpayers in the wage distribution, 0.06 *more* were in the top 0.5 per cent of capital income receivers in the US in 2000 than in Norway in 2000.

Table 7 Difference in survival function for capital and wage income Norway 2009 compared with 1993

	Whole	Top half	Top 40 per cent	Top 20 per cent	Top 10 per cent	Top 5 per cent	Top 1 per cent	Top 1/2 per cent
Up to P50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P60	0.00	1.32	0.89	0.29	0.08	0.10	0.04	0.02
P80	0.00	1.19	1.01	0.55	0.27	0.08	0.05	0.03
P90	0.00	0.62	0.80	0.76	0.43	0.16	0.03	0.01
P95	0.00	0.28	0.41	0.46	0.25	0.07	-0.01	-0.02
P99	0.00	0.13	0.20	0.23	0.11	0.00	-0.04	-0.03
P99.5	0.00	0.02	0.03	0.03	-0.01	-0.04	-0.06	-0.03
P100	0.00	0.01	0.01	0.01	-0.02	-0.04	-0.05	-0.04

Notes: (1) the numbers may not add up exactly on account of rounding; (2) the shading shows the cells that are negative by more than rounding error.
Reading: of the top 0.5 per cent of taxpayers in the wage distribution, 0.04 *fewer* were in the top 5 per cent of capital income receivers in 2009 than in 1993.

Figure 1 Effect on integrated survival copula of a pair of diagonalizing switches in opposite directions



Note: A diagonalizing switch of size δ at (I,j) and of –δ at (s,t)